Invariance of selected Pareto-optimal multi-attribute information systems for various application environments in designated subspace of the weight space

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Using a widely known Weighted Sum Method (WSM) for choosing an information system (IS) from a finite set of candidate multi-attribute information systems to be deployed in an information systems environment, this paper demonstrates that the selected Pareto-optimal IS remains the same provided that the weights remain inside a specified subspace of the weight space. Three cases of IS Environments are discussed and analyzed. The invariance properties hold for other information systems environments if there is a finite number of candidate information systems to be deployed in an IS environment. There is no previous literature describing invariance of the selected Pareto-optimal multi-attribute information system, when weights are inside a designated subspace of the weight space.

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Email Address: cruz.22@osu.edu
Date received: June 16, 2019
Date revised: October 01, 2019
Date accepted: October 17, 2019

KEYWORDS

information systems environment, multiple-attribute information systems, Pareto-optimality, weighted sum of performance criteria

INTRODUCTION

Information Systems (IS) are computer-based hardware and software systems deployed in application environments. Information Systems are studied in degree programs such as Computer Science, Information Technology and Information Systems. Of these three programs, the degree program Bachelor of Science in Information Systems seems to be least understood. The BSIS program typically includes courses in “fundamentals and applied practice in applications development, data and information management; information technology infrastructures; systems analysis, design and acquisition; project management and the role of information systems in organizations.” (ABET Criteria for Accrediting Computing

Programs, 2019). The Information systems are deployed in IS Environments.

What is an Applications Environment or an IS Environment? ABET defines an Information Systems Environment as “an organized domain of activity. Information systems are used to support and enable the goals of the activity. Examples of IS Environments include (but are not limited to) business, healthcare, government agencies, not-for-profit organizations, and scientific disciplines.” (ABET Criteria for Accrediting Computing Programs, 2019). One of the Student Outcomes for the BSIS program is to “support the delivery, use, and management of information systems within an IS Environment.” (ABET Criteria for Accrediting Computing Programs, 2019).

How are the information systems selected? Section 2.1 briefly reviews some methods for doing this. This paper utilizes a well-known method, the Weighted Sum Method (WSM), a weighted sum of the criteria, for selecting an information system to be deployed in an IS Environment when the information system has multiple attributes. To the best of the authors’ knowledge there is no previous work on describing the invariance of the selected Pareto-optimal multi-attribute information system for deployment in an IS Environment, when the weights are inside designated subspaces of the weight space.

The relevant and important attributes to be considered in choosing a multi-attribute information system may be different for different application environments. For each IS environment, various multiple attributes are first chosen. For a given IS environment, several different information systems are considered, each characterized by the attributes selected for the application environment. Each of the IS candidates is rated for each of the selected attributes. Finally, an information system is selected from the several candidate IS. This selection task is the core of decision making in complex situations.

Developing a software application involves a tough decision of selecting from a finite number of acceptable software applications, each satisfying system requirements. The selection is not simple because there might not be a unique choice that possesses advantage for each of the attributes. One suggested approach to this challenge (Blaich and Wise 2018) mentioned that a classic conceptual tool for project managers is the Iron Triangle whose thrust is dictated by how one prioritizes three factors: scope, cost, and speed. They mentioned that the Iron Triangle is a way of reminding clients that one cannot have it all when it comes to implementing a project, one could just pick two, whether it is big, fast, or cheap. However, they do not describe a procedure for making a choice. Recent procedures for making a selection are reviewed in Section 2.1.

METHODOLOGY

The Information Systems to be deployed in various IS environments have multiple attributes, in realistic cases. These attributes are also performance criteria. In practical applications we consider choosing an IS from a finite (greater than two) set of potential information systems, each with multiple attributes, to be deployed in an IS environment. When one IS is dominant (Cruz and Almario 2018), the choice is clearly the dominant IS. When there is no dominant IS, in general, some IS might have better attributes than other options, but at the same time, these IS have other attributes that are worse than those of other IS, and choosing an IS is no longer as clear (Pareto 1897, Zadeh 1963). In this case, there are at least two Pareto-optimal IS, following from the result in (Cruz and Almario 2018). In this paper, it is assumed that there is no dominant IS among the candidate IS (as the selection problem is very simple in this case) but there might be some dominated candidate IS

Multiple Criteria Decision Making (MCDM) Literature Review

The general problem of optimizing multiple criteria has a long history, dating back to Pareto (Pareto 1897). In this brief literature review we cite some of the widely referenced books and papers. Among the early seminal papers are by ones by Zadeh, Eckenrode, Fishburn, and Geoffrion (Eckenrode 1965, Fishburn 1967, Geoffrion 1968, Zadeh, 1963). Popular book references are by Chankong and Haimes (Chankong and Haimes1983), Saaty (Saaty 1980), Sawaragi (Sawaragi et al 1985) and Ehrgott (Ehrgott 2005). In these references, the multiple criteria, are combined into a scalar function, the most common being the weighted sum of the criteria (Eckenrode 1965, Marler and Arora 2010), also known as the Weighted Sum Method (WSM). In these references the number of options could be infinite, and in most cases, it is a continuum. The set of options may contain dominated options. Optimizing the weighted sum will always yield a Pareto-optimal option (Zadeh 1963). The set of Pareto-optimal options is usually a continuum if the set of weights is a continuum. When the number of options is finite there are at least two Pareto-optimal options (Cruz and Almario 2018). There are other scalarization methods besides the WSM, but these might not yield a Pareto optimal option.

It is usually desirable to perform a sensitivity analysis of the selected option with respect to small changes in the weights. For example, the partial derivative of the optimum weighted sum of criteria with respect to the weights might be calculated. This is called a sensitivity function (Cruz 1973). In general, when the options are in a continuum, any change in the weights will result in a different Pareto-optimal option to be chosen. The partial derivative of the weighted sum with respect to the weights is generally not zero. When the number of options is finite, Cruz and Almario (Cruz and Almario 2018) showed that there exists a continuum of subspaces of the weight space such that the selected Pareto-optimal option remains the same. In this case, the partial derivative of the weighted sum with respect to the weight is zero, in an entire subspace of the space of continuum weights. Thus, the selected Pareto-optimal choice has invariance properties with respect to changes in the weights. Cruz and Almario (Cruz and Almario 2018) are apparently the first in the literature to show this property, when the set of options is finite. Tan (Tan et al 2019) provide a procedure to calculate the invariance subspace.

What about the choice of an IS in an IS environment? There are recent papers describing methods for selecting the IS in an IS environment. Krmac and Djordjevic (Krmac and Djordjevic 2019) describe a method for determining the normalized weights in the WSM, using Strengths, Weaknesses, Opportunities, Threats (SWOT) analysis and Analytic Hierarchical Process (AHP) of Saaty (Saaty 1980) for the evaluation of Train Control Information Systems (TCIS). The TCIS is for the European RailwayTraffic Management System (ERTMS) IS environment (Parent de Curzon 1999). In Erdogan (Erdogan et al 2019), the weights in the WSM are determined for a cyber security IS in a security environment using a fuzzy based methodology. The cyber security IS involved four criteria and 10 candidate IS. In these two references, the focus is on determining weights for a WSM. There was no sensitivity analysis with respect to changes in the weights. In Daghouri (Daghouri et al 2018), several methods (including the WSM) for selecting an IS for an IS environment are compared. They conclude that with one exception the final choices are the same. From Zadeh (Zadeh 1963) the preponderant identical answer must be a Pareto-optimal choice, and the exception must not be Pareto-optimal; hence it must be a dominated selection. This fact is not noted in Daghouri (Daghouri et al 2019).

This paper is an application of a methodology (Cruz and Almario 2018) to the area of choosing an information system from a finite set of multi-attribute information systems, for deployment in an IS Environment. The procedure is a detailed application of the WSM. First, the Performance Criteria are all converted to standardized values, using the scaling, shifting, and normalization method in (Cruz and Almario 2018). Next, weights are assigned to the various performance criteria. The choices of weights are subjective, but the AHP method of (Saaty 1980) may be utilized, and a weighted sum of the standardized criteria is calculated for each information system. The highest weighted sum among the finite number of IS candidates is chosen as the winning information system. As shown in (Cruz and Almario 2018) the winning option (the winning information system in this paper) is either a dominant option or a Pareto optimal option, as guaranteed by Zadeh (Zadeh 1963). The winning information system has various invariance properties (Cruz and Almario 2018). "The winning option remains invariant with respect to a range of weights provided that the weights remain in specified subspaces. Although the choices for the weights are subjective, the methodology provides "additional confidence about the chosen weights and the resulting winning option." (Cruz and Almario 2018). ***The result on the existence of an invariant subspace of the weight space for the selection of the winning Pareto-optimal IS in an IS environment is new and there is no previous literature on this topic.***

The methodology is applied to three different IS Environments in the following section. In subsection 1, a multi-attribute information system is chosen for a Business Environment. In subsection 2, an IS system is chosen for a Library Management System Environment. In subsection 3, an IS system is chosen for a Building Security Environment.

SOME INFORMATION SYSTEMS ENVIRONMENTS

1. ENTRANCE ADMISSION SYSTEM FOR A BUSINESS ENVIRONMENT

Consider a Business IS Environment, A component of the information system plan is the employee entrance admission system. Biometric technologies are being considered in the employee entrance admission application development.

Biometrics is a method for personal authentication, which uses individual information from a person's face, fingerprint, iris, palmprint, or gait. Biometrics cannot be stolen, forgotten, or shared. And biometrics provides a greater degree of security compared with traditional authentication methods (Jeong, et. al. 2006).

Table 1 shows three different biometrics technologies we selected from a literature survey: fingerprint recognition, facial recognition, and iris recognition. The four multiple attributes or performance criteria that we chose to help the client in selecting what biometrics technology to use are also shown in Table 1: Cost, False Acceptance Rate (FAR), False Rejection Rate (FRR), and Market Share.

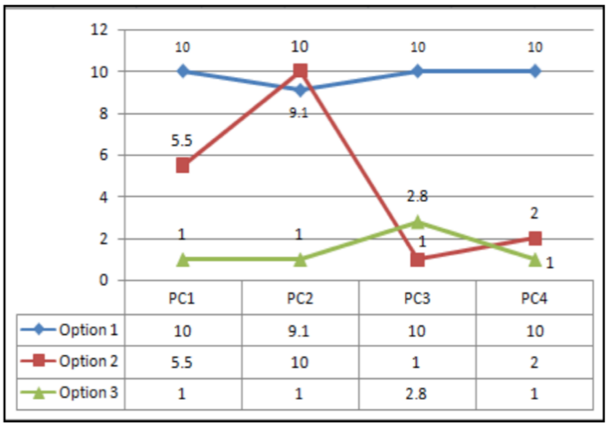
Table 1: Three Biometric Technologies for Employee Attendance Monitoring System for a Business Environment, Each with Four Attributes

Option	Biometric	Cost	FAR	FRR	Market Share (Thakkar, 2018)
1	Finger print recognition	P5,000	0.01%	0.6 %	31%
2	Facial recognition	P10,000	0.0%	1.1%	15%
3	Iris recognition	P15,000	0.1 % (ICE 2006 cited by Thakkar, 2018)	1% (ICE 2006 cited by Thakkar, 2018)	13%

The cost of the biometric technology is priced in Philippine Peso currency. A lower cost is preferable to a higher cost. FAR, in biometrics, is the probability of a security system incorrectly verifying or identifying an unauthorized person. A lower value of FAR is preferable to a higher value of FAR. False Rejection Rate (FRR) is the probability of a security system failing to verify or identify an authorized person. A lower value of FRR is preferable to a higher value of FRR. Values of FAR and FRR were taken from Fingerprint Vendor Technology Evaluation 2003 (Wilson, et.al. 2004) as mentioned by Thakkar (2018) and study conducted by Batinggal and Niguidula (2018). Performance criterion 4 is the Market Share where it shows the statistics of users deploying biometrics with data taken from Thakkar (Thakkar 2018). A higher value of Market Share is preferable to a lower value of Market Share.

Option 1 offers fingerprint recognition whose hardware cost in the market is P5,000. According to studies, the FAR of fingerprint recognition is 0.01% while its FRR is 0.6%. Many enterprises are using it, having 31% of the market share compared to other biometrics applications. Option 2 supports facial recognition technology. The hardware cost is P10,000.00, and FAR rate of facial recognition is 0.0%, while FRR is 1.1% . The reported market share was 15%. Option 3 uses iris recognition technology where average hardware cost is P15,000. The FAR and FRR of iris recognition are 0.1% and 1% respectively. The market share was reported at 13%.

Table 1 consists of different raw values of the attributes (Performance Criteria), without normalization. The shifted, scaled and normalized values, or standardized values of the multiple attributes for the various options are depicted in Figure 1, in tabular form (using transformation equations in Cruz and Almario 2018) and in graphical form.



Note: PC1: Cost, PC2: FAR, PC3: FRR, PC4: Market Share

Figure 1: Multiple Attributes, with Standardized Values, of Biometric Technologies Options for Entrance Admission System, in a Business Environment, Shown in Graphical and Tabular Forms.

Option 3 is dominated by option 1 (See definition in Cruz and Almario 2018). Option 3 will never be chosen. Options 1 and 2 are Pareto optimal (Cruz and Almario 2018). It may seem that visually, option 1 would be preferable to option 2, but option 2 is better than option 1 for PC2. The winning option would depend on the weights we use for the four Performance Criteria. PC2 is the probability of a false acceptance rate (FAR) or the probability that the recognition system accepts an unauthorized person as an employee. Depending on how much more important PC2 is compared to PC 1, PC 3, and PC 4, option 2 could be a winner. For example, if we use a weight of 100% for PC 2, 0% for PC 1, 0% for PC 3, and 0% for PC 4, option 2 would win. Depending on the nature of the business environment, any unauthorized entry might be a disaster. How close to 100% does the weight for PC 2 have to be for option 2 to be a winner?

Suppose x is the weight of PC 2. Let the weight for PC 1, PC 3, and PC 4 be
$(1/3)(1-x)$.

In order for option 2 to be better than option 1, the weighted sum of the standardized performance criteria for option 2 should be greater than the weighted sum of the standardized performance criteria for option 1,
$(1/3)(1-x)(5.5+1+2) + (x)(10) > (1/3)(1-x)(10+10+10) + 9.1x$.
Simplifying, we get $x > 21.5/24.2 = 88.84\%$.

Similarly, in order thatr option 2 be better than option 3,

$(1/3)(1-x)(5.5+1+2) + (x)(10) > (1/3)(1-x)(1+2.8+1) + 1.0(x)$.

Simplifying we get

$x > -3.7/23.3$

In order that option 2 be better than both options 1 and 3 for all weights x for PC 2 (FAR),
the weight x needs to be in the range

$0.8884 < x < 1.00$,

and option 2 would remain to be the winning option. For

$0 < x < 0.8884$

option 2 would not win. At this point, option 1 or option 3 could win. This would make sense if FAR does not have very serious consequences.

In order that option 1 be better than option 3, in addition to $x < 88.84\%$, we need

$(1/3)(1-x)(10+10+10) + 9.1x > (1/3)(1-x)(1+2.8+1) + 1.0(x)$.

Simplifying, we obtain

$9.1x > -(1/3)(1-x)(25.2) + x$ or

$x < 28$.

We conclude that for

$0 < x < 0.8884$

option 1 would be the winning option.

Figure 2 shows the graph of the weighted sums of the standardized performance criteria for option 1, option 2, and option 3, for different values of x, (the weight for standardized criterion 2), for $0 < x < 1$. For $0 < x < 0.8884$, option 1 is the winning choice, and for $0.8884 < x < 1$, option 2 is the winning choice, as analyzed earlier. As concluded earlier, option 3 is dominated by option 1. This is clear from the graphs in Figure 2.

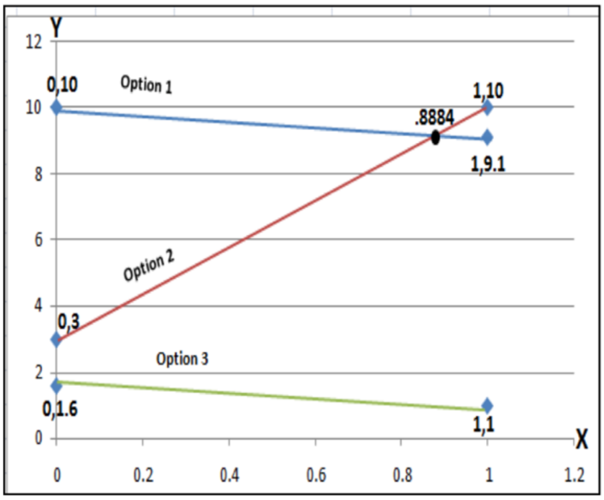


Figure 2: Graph of the weighted sum of standardized criteria for options 1, 2, and 3, for the business environment plotted against the weight x for criterion 2, the False Acceptance Rate.

The client would be advised on the consequences of choosing the weight x for FAR. For example, if FAR should be very low, the weight for PC 2 might be chosen as 94% leading to a winning option 2, the facial recognition technology, with a FAR = 0%. This winning option would be invariant inspite of variations in the weight for PC 2 of 94% from 88.84% to 100%. This invariance property follows from one of the results in (Cruz and Almario 2018).

Trade-Off of Multiple Attributes

When the normalized weight x of the False Acceptance Rate (FAR) is in the range $0 < x < 0.8884$, Option 1 is optimum. It has the least cost, the least False Rejection Rate (FRR), and the highest Market Share. However, it traded off these superior attributes with a degradation of FAR. When x is in the range $0.8884 < x < 1$, Option 2 is optimum. It has the best FAR but at a higher cost, higher False Rejection Rate (FRR), and lower market share. The quantitative amounts of the trade-offs can be seen in Table 1.

2. BOOKS IDENTIFICATION SYSTEM IN A LIBRARY MANAGEMENT SYSTEM ENVIRONMENT

Consider a Library System Management IS Environment. As a trend, many customized digital libraries are being developed for creating, storing, and making the content available for the users. Publishing houses are also developing and using software for preservation and access to their publications (Arora, 2018). A client would like to have a fully automated Library Management System. As stated in the Library Information Systems Plan, one priority area is the automation of books identification in the library. Table 2 shows the options that can be offered in the development of the books identification system.

Table 2: Technologies Available for Books Identification System

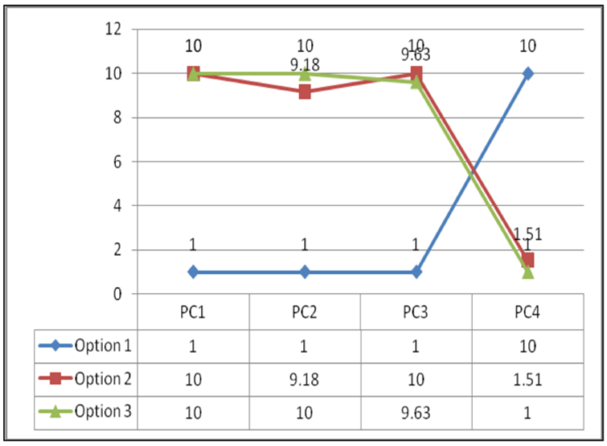
Option	Technology	Printer Price	Scanner Price	Cost per Tag	Read Range
1	RFID	$1,500	$1,500	$0.05 to $1.00	30 ft
2	Barcode	$400	$500	$0.01	2.5ft
3	QR Code	$400	$400	$0.05	0.833ft

Table 2 shows the possible technologies that can be used in identifying books in the library. These are QR Codes, Barcodes, and RFID (radio frequency identification). The attributes or performance criteria chosen for this application environment are printer price, scanner price, cost per tag and read range. The costs that can be seen in the table are in US $ currency and the read range is in feet or inches. Lower costs are preferable to higher costs for Performance Criteria 1, 2 and 3. A longer range is preferable to a shorter range for Performance Criterion 4. Option 1 offers RFID with printer and scanner price that costs $1,500 each. Cost per tag ranging from $0.05 to $1.0 and the scan range could reach 0.833 ft or more depending on the model. Option 2 offers Barcode where average price of the printer costs $400 while average reader can cost $500. The price per tag is just $.01 and range could reach 2.5ft or more. Option 3 offers QR Code whose reader and printer's average cost is $400 each, the average read range of QR code is 0.833ft depending on its size.

The data in Figure 3 are the scaled, shifted and normalized data of Table 2, using equations of Cruz and Almario 2018, depicted in graphical form and tabular form. There are no dominated options and there is no dominant option. All three options are Pareto optimal. Each of the three Information Systems has four attributes. The Information System is to be deployed in an automated library environment. Information System 1 is more costly than the two other systems, but it has the farthest read range. The cost per tag is also highest. To apply the method for choosing among Pareto options in (Cruz and Alamario 2018), we need to assign weights to the four PCs.

For example, suppose read range is most important so that weight for PC 1 is $w1 = .02$, weight for PC 2 is $w2 = .02$, weight for PC 3 is $w3 = .02$ and weight for PC 4 is $w4 = .94$,
The weighted sum for option 1 is
$0.02(1) + 0.02(1) + 0.02(1) + 0.94(10) = 9.46$
The weighted sum for option 2 is
$0.02(10) + 0.02(9.18) + 0.02(10) + 0.94(1.51) = 2.008$
The weighted sum for option 3 is
$0.02(10) + 0.02\ (10) + 0.02(9.63) + 0.94(1) = 1.5326$
Hence the winning option is option 1, the RFID technology, where the weights are 0.02, 0.02, 0.02, and 0.94.

More generally, suppose we set the weight for PC 4 as x and the weights for the other three PCs as $(1/3)(1 – x)$ each.
In order that RFID be the winning option, the weighted sum for option 1(RFID) should be
greater than the weighted sum for option 2 (Bar Code)
$(1/3)(1 – x)(1 + 1 + 1) +x(10) > (1/3)(1 – x)(10 +9.18 + 10) + x(1.51)$
and the weighted sum for option 1 should be greater than the weighted sum for option 3 (QR Code)
$(1/3)(1 – x)(1 + 1 + 1) +x(10) > (1/3)(1 – x)(10 + 10 + 9.63 + x(1)$.



Note: PC1: Printer Price, PC2: Scanner Price, PC3: Cost per Tag, PC4: Range

Figure 3: Multiple Attributes with Standardized Values, of Technologies Options for Books Identification System, in a Library Management System, Shown in Graphical and Tabular Forms.

Simplifying,
$9x + 1 > 9.727(1 – x) + 1.51x$ or $17.217x > 8.727$ and
$9x + 1 > (1/3)(1 – x)(10 + 10 + 9.63) + x(1)$ or $17.877x > 8.877$.
$x > 0.507$ and $x > 0.497$. Both inequalities would be satisfied for a weight for RFID of $x > 0.507$
and the weights for the other three PCs set to be equal to $(1/3)(1 – x)$
The RFID technology will be the winning option for $0.507 < x < 1.00$.

Suppose that the client initially selected a weight of 75% for the read range Performance Criterion, thus ending with a recommended RFID technology option. This winning option is invariant, inspite of variations in the weight of 75% anywhere from 50.7% to 100%. This invariance property follows from one of the results in (Cruz and Almario 2018).

What happens if the weight x is set to less than 0.507? Then the RFID option 1 will no longer be the winning option. When would option 2 be a winning option? Option 2 would be a winning option if the weighted sum for option 2 is greater than the weighted sum for option 3, in addition to $x < 0.507$. Thus $(1/3)(1 – x)(10 +9.18 + 10) + x(1.51) > (1/3)(1 – x)(10 + 10 + 9.63 + x(1)$.

Simplifying we have $x > 0.2273$. We conclude that Option 2 would be a winning option when
$0.2273 < x < 0.507$.
Finally, option 3 would be the winning option for
$0 < x < 0.2273$

Figure 4 graphically displays the weighted sums of the standardized performance values for the three technology options for Books Identification Systems. Notice that option 2 is the winning option for $0.2273 < x < 0.507$, option 1 is the winning option for $0.507 < x < 1$, and option 3 is the winning option for $0 < x < 0.2273$, where x is the weight for the read range (PC 4).

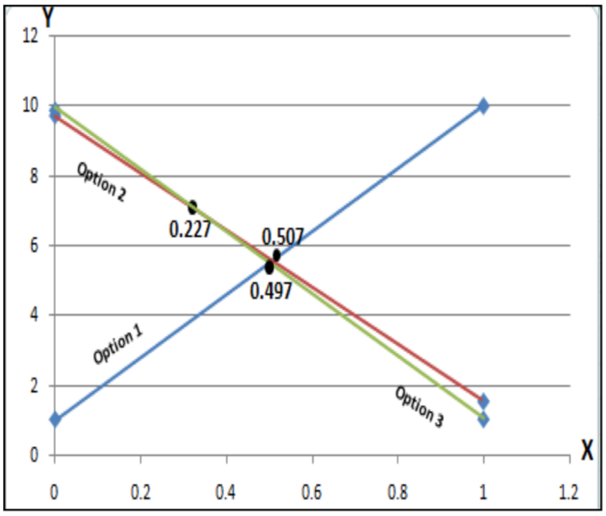


Figure 4: Graph of the weighted sum of standardized criteria for options 1, 2, and 3, for the automated library environment plotted against the weight x for criterion 4, the read range

Trade-Off of Multiple Attributes

When the normalized weight x for PC 4 is in the range $0 < x < 0.2273$, Option 3 is optimum. It has the least printer price, the least scanner price, a slight increase in cost per tag, the shortest scan range (worst). It achieved the best printer price and the best scanner price in exchange for a degradations of cost per tag and scan range. When x is in the range $0.2273 < x < 0.507$, Option 2 is optimum. Option 2 has the best printer price and the best cost per tag in exchange for some degradation in scanner price and the worst scan range. When $0.507 < x < 1$, the optimum option is Option 1. Option 1 has the best scan range in exchange for degradations in printer price, scanner price, and cost per tag. The quantitative amounts of the trade-offs can be seen in Table 2.

3. CLOSED CIRCUIT TELEVISION SYSTEM IN A BUILDING SECURITY SYSTEM ENVIRONMENT

Consider a Building Security IS Environment. One of the elements of a Building Management System is the Safety and Security System, which could include presence of security personnel and use of technology such as Closed-Circuit Television (CCTV). Consider three possible closed-circuit television systems. as indicated in Table 3.

For this application environment, seven attributes or Performance Criteria are selected. Performance Criterion (PC) 1 is camera price in US dollars. PC 2 is camera resolution in pixels. PC 3 is Number of cameras. PC 4 is number of Digital Video Recorder (DVR) channels. PC 5 is DVR size in terabytes. PC 6 is DVR price in US dollars. PC 7 is size of the TV monitor in inches. For PC 1 and PC 6, lower cost is preferable to higher cost. For PC 2 higher resolution is preferable to lower resolution. For PC 3, more cameras are preferable to fewer cameras. For PC 4 more channels are preferable to fewer channels. For PC 5 more terabytes is preferable to fewer terabytes. For PC 7 bigger size monitor is preferable to smaller size monitor.

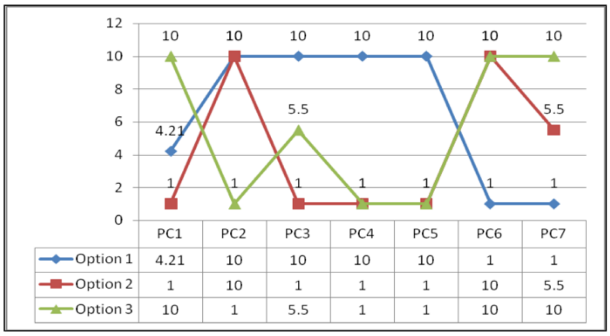
Option 1 will place 8 cameras with 1080p resolution. It also has 16 channels DVR that could accommodate additional camera in the future. Camera price is $75 while DVR price is $500. The package includes a 32in television monitor to view the 8 cameras simultaneously. Option 2 offers only 6 cameras capable of securing the building with 1080p resolution. The DVR comes with 8TB storage and a 40-inch television monitor for better viewing of camera footage. Option 3 has 7 cameras with 720p resolution. It comes with 8 channels DVR with 8 TB storage capacity and 48 inches television.

The data in Table 3 are scaled, shifted and normalized to their standardized values, and the standardized data are show in Figure 5 depicted graphically and in tabular form, using equations of (Cruz and Almario 2018) based on the raw values in Table 3.

Table 3: Table 3: Specifications for CCTV Components for Building Security System

Option	Price / Camera	Camera Resolution	Number of Cameras	DVR Channels	DVR Size	DVR Price	Monitor size (in)
1	$75	1080p	8	16 channel	16 TB	$500	32
2	$85	1080p	6	8 channels	8TB	$350	40
3	$56.99	720p	7	8 channels	8TB	$350	48

In order to proceed with choosing a winning option among the three options we assign weights to the seven Performance Criteria, in consultation with the client. Suppose that it is agreed that camera resolution and monitor size are much more important than the other 5 PCs. So, suppose that weight for PC 2 is 37.5%, weight for monitor size is 37.5% and the weights for the other five PCs is 5% each.



	PC1	PC2	PC3	PC4	PC5	PC6	PC7
Option 1	4.21	10	10	10	10	1	1
Option 2	1	10	1	1	1	10	5.5
Option 3	10	1	5.5	1	1	10	10

PC1: Camera Price, PC2: Camera Resolution, PC3: Number of Cameras, PC4: DVR Channels, PC5: DVR Size, PC6: DVR Price, PC7: Monitor Size

Figure 5: Multiple Attributes, with Standardized Values, of Closed-Circuit Television Options, in a Building Security System Environment, Shown in Graphical and Tabular Forms.

The weighted sum for option 1 is
(0.05)(4.21 + (0.375)(10) + 0.05(10) + 0.05(10) + 0.05(10) + 0.05(1) + 0.375(1) = 5.8855
The weighted sum for option 2 is
0.05(1) + 0.375(10) + 0.05(1) + 0.05(1) + 0.05(1) + 0.05 (10) + 0.375(5.5) = 6.5125
The weighted sum for option 3 is
0.05(10) + 0.375 (1) + 0.05(5.5) + 0.05(1) + 0.05(1) + 0.05(10) + 0.375(10) = 5.5

The weighted sum for option 2 is largest, thus option 2 is the winning option. Option 2 offers 8 cameras capable of securing the building with 1080p resolution. The DVR comes with 8TB storage and a 40-inch television monitor for large viewing of camera footage. Option 2 would remain as the winning option for small changes in the 7 weights, based on one of the invariance properties described in (Cruz and Almario 2018).
More generally, suppose that the weight for PC 2 is x, the weight for PC 7 is x, and the weight for PC1, PC 3, PC 4, PC 5, and PC 6 is (1/5)(1 – 2x) each. Then the weighed sums of the three standardized PCs are:

For option 1,
(1/5)(1 – 2x)(4.21 + (x)(10) + (1/5)(1 – 2x) (10) +(1/5)(1 – 2x) (10) + (1/5)(1 – 2x) (10) + (1/5)(1 – 2x) (1) + x(1) = 7.042(1 – 2x) + 11x = 7.042 – 3.084x.

For option 2,
$(1/5)(1 - 2x)(1 + 1 + 1 + 1 + 10) + (x)(10) + (x)(5.5) = 2.8(1 - 2x) + 15.5x = 9.9x + 2.8$

For option 3,
$(1/5)(1 - 2x)(10 + 5.5 + 1 + 1 + 10) + (x)(1 + 10) = 5.5(1 - 2x) + 11x = 5.5$.

The weighted sum of standardized PCs for the three options are plotted in Figure 6 from $x = 0$ to $x = 1.0$, where x is the weight for PC2, x is also the weight for PC 7 and $(1/5)(1 - 2x)$ is the weight for each of PC1, PC 3, PC 4, PC5, and PC 6. From Figure 6, option 1 is the preferred option for
$0 < x < 0.3267$
and option 2 is the preferred option for
$0.3267 < x < 1.0$

Figure 6 displays the weighted sum of the seven standardized values of PCs, for the three options for CCTV. As analyzed above, option 1 is the preferred choice for $0 < x < 0.3267$, and option 2 is the preferred choice for $0.3267 < x < 1$. Although option 3 has the largest monitor, it is not chosen for any value of x, which is the weight for PC 2 (resolution), and also the weight for PC 7, (monitor size).

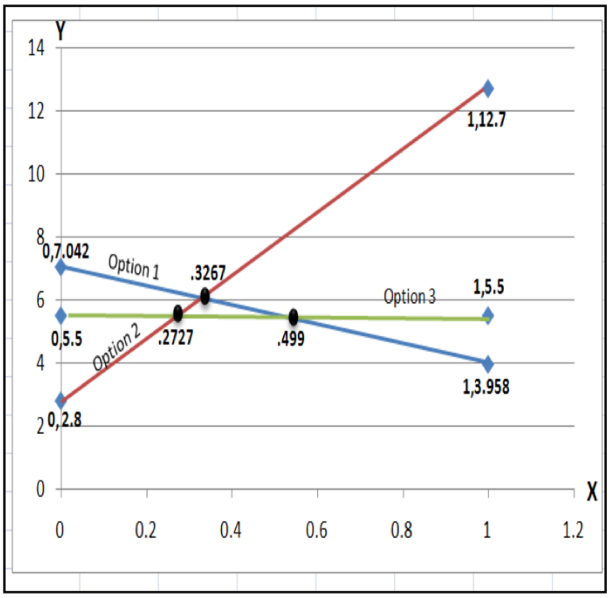


Figure 6: Graph of the weighted sum of standardized criteria for options 1, 2, and 3, for the building security environment plotted against the weight x for criterion 7, the monitor size.

Trade-Off of Multiple Attributes

When the normalized weight x for PC 2 (the same as the weight for PC 7) is in the range $0 < x < 0.3267$, Option 1 is optimum. It has the best values for PC 2, PC 3, PC 4, and PC 5 It achieved these benefits in exchange for degradations of PC 1, PC 6, and PC 7. When x is in the range $0.3267 < x < 1$, Option 2 is optimum. Option 2 has the best values for PC 2, and PC 6, in exchange for degradations in PC 1, PC3, PC 4, PC 5, and PC 7 The quantitative amounts of the trade-offs can be seen in Table 3.

INVARIANT SUBSPACES

In the three Information Systems of the three sections above, we displayed the intervals of a one-dimensional subspace for invariance in the selected winning options. This was possible by restricting the other dimensions to be dependent on the chosen dimension. For example, in Section 3, the normalized weight for PC 2 was chosen as x but the other three weights were restricted to be $(1/3)(1 - x)$. There is always a constraint that the sum of the various normalized weights should be 1.0. Thus, since there are four normalized weights there should be three remaining independent normalized weights in the most general case. In Section 4, we chose the normalized weight for PC 4 as x and constrained the other three normalized weights to be $(1/3)(1 - x)$. As in Section 3, there are four normalized weights so there should be three independent normalized weight remaining, in general. Finally, in Section 5, we chose the normalized weight for PC 2 to be equal to the normalized weight for PC 7, both equal to x, and then we set the remaining five other normalized weight to each to be equal to $(1/5)(1 - 2x)$. In the most general case, there should be six remaining independent normalized weights. With the simplifying assumptions in Sections 3, 4, and 5 we can display two-dimensional graphs in Figures 2, 4, and 6, for deeper visual clarity and insight. In Cruz and Almario (Cruz and Almario 2018) the general invariant subspaces are clearly specified by a set of linear inequalities in m-dimensional space, (a convex hyper polyhedron) where m is the number of attributes. No specific numerical procedure was suggested in Cruz and Almario (Cruz and Almario 2018) to obtain the invariant subspace. Recently, a procedure for numerically obtaining the general invariant subspace was provided (Tan et al 2019).

The description and analysis of the invariance of the selected Pareto-optimal information system that is deployed in an IS environment when the weights remain in a specified subspace of the weight space is revealed for the first time in this paper. To the best of the authors' knowledge there is no previous literature on this topic.

CONCLUSION

Using a widely known WSM for choosing an information system from a finite set of candidate multi-attribute information systems to be deployed in an information systems environment, we demonstrated that the selected Pareto-optimal IS remains the same if the weights are inside a specified subspace of the weight space. Three cases of IS Environments were described.. The invariance properties hold for other information systems environments provided that there is a finite number of candidate information systems to be deployed in each IS environment.

ACKNOWLEDGEMENT

The work of Jose B. Cruz, Jr. was supported by a Research Fellowship from the National Academy of Science and Technology of the Philippines (NAST PHL). He is an Academician of NAST PHL.

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